

SEPARATION OF BRANCHES OF $O(N-1)$ -INVARIANT SOLUTIONS FOR A SEMILINEAR ELLIPTIC EQUATION

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ABSTRACT. We consider the problem

$$\begin{cases} -\Delta u = u^p + \lambda u & \text{in } A \\ u > 0 & \text{in } A \\ u = 0 & \text{on } \partial A \end{cases}$$

where A is an annulus in \mathbb{R}^N , $N \geq 2$, $p \in (1, +\infty)$ and $\lambda \in (-\infty, 0]$. Recent results, [GGPS], ensure that there exists a sequence $\{p_k\}$ of exponents ($p_k \rightarrow +\infty$) at which a nonradial bifurcation from the radial solution occurs. Exploiting the properties of $O(N-1)$ -invariant spherical harmonics, we introduce two suitable cones \mathcal{K}^1 and \mathcal{K}^2 of $O(N-1)$ -invariant functions that allow to separate the branches of bifurcating solutions from the others, getting the unboundedness of these branches.

1. INTRODUCTION

In this paper we consider the problem

$$(1.1) \quad \begin{cases} -\Delta u = u^p + \lambda u & \text{in } A \\ u > 0 & \text{in } A \\ u = 0 & \text{on } \partial A \end{cases}$$

where A is an annulus of \mathbb{R}^N , i.e. $A := \{x \in \mathbb{R}^N : a < |x| < b\}$, $b > a > 0$, $N \geq 2$, $p \in (1, +\infty)$ and $\lambda \in (-\infty, 0]$. For simplicity one can think to (1.1) with $\lambda = 0$.

It is well known that problem (1.1) has a radial solution for any $p \in (1, +\infty)$ (see [KW]), and that this solution is unique if $\lambda \in (-\infty, 0]$ (see [T] and [FMT]). We will denote by u_p this radial solution and by \mathcal{S} the curve of radial solutions of (1.1) in the product space $(1, +\infty) \times C_0^{1,\alpha}(\overline{A})$, where $C_0^{1,\alpha}(\overline{A})$ is the set of continuous differentiable functions on \overline{A} which vanish on ∂A and whose first order derivatives are Hölder continuous with exponent α . In other words:

$$(1.2) \quad \mathcal{S} := \{(p, u_p) \in (1, +\infty) \times C_0^{1,\alpha}(\overline{A}) \text{ such that } u_p \text{ is the radial solution of (1.1)}\}.$$

In this paper we study the nonradial solutions that bifurcate from the curve \mathcal{S} as the exponent p varies. Let us recall that a point $(p_k, u_{p_k}) \in \mathcal{S}$ is a *nonradial bifurcation point* if in every neighborhood of (p_k, u_{p_k}) in $(1, +\infty) \times C_0^{1,\alpha}(\overline{A})$ there exists a nonradial solution (p, v_p) of (1.1).

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In the paper [GGPS] the authors show that there exists a sequence of values of the exponent p_k such that $p_k \rightarrow +\infty$ and (p_k, u_{p_k}) is a nonradial bifurcation point for \mathcal{S} .

These values p_k are found considering the linearized equation at the radial solution u_p . In [GGPS] it is shown that u_p is degenerate if and only if, for some $k \geq 1$,

$$(1.3) \quad \alpha_1(p) + k(N - 2 + k) = 0$$

where $\alpha_1(p)$ is the first eigenvalue of the one-dimensional operator

$$(1.4) \quad \widehat{L}_p(v) = r^2 \left(-v'' - \frac{N-1}{r}v' - pu_p^{p-1}v - \lambda v \right)$$

in the space of functions of $H_0^1(a, b)$. From the analyticity of $\alpha_1(p)$ with respect to p and from the asymptotic behavior of $\alpha_1(p)$ as $p \rightarrow 1$ and as $p \rightarrow +\infty$ it is proved in [GGPS] that for any $k \geq 1$ the quantity $\alpha_1(p) + k(N - 2 + k)$ changes sign as p varies in $(1, +\infty)$. Each time $\alpha_1(p) + k(N - 2 + k)$ changes sign an eigenvalue of the linearized operator changes sign so that the Morse index of the radial solution changes. Let us call *Morse index changing points* the pairs $(p_k, u_{p_k}) \in \mathcal{S}$ such that the Morse index of the radial solution u_p changes at p_k . These points are characterized by

$$(1.5) \quad (\alpha_1(p_k + \delta) + k(N - 2 + k))(\alpha_1(p_k - \delta) + k(N - 2 + k)) < 0 \quad \text{for } \delta \in (0, \delta_0)$$

for some $\delta_0 > 0$ and for some $k \geq 1$. In [G] it is shown that if (p_k, u_{p_k}) is a Morse index changing point there exists a continuum, $\mathcal{C}(p_k) \subset (1, +\infty) \times C_0^{1,\alpha}(\overline{A})$, of nonradial solutions bifurcating from that point. This continuum $\mathcal{C}(p_k)$ obeys the so-called Rabinowitz alternative, (see Theorem 3.3 in [G]), i.e. either $\mathcal{C}(p_k)$ is unbounded in $(1, +\infty) \times C_0^{1,\alpha}(\overline{A})$ or it must meet the curve of radial solutions \mathcal{S} in another Morse index changing point. Here we are able to prove that the second alternative is not possible when $k = 1$ or 2 and $N \geq 3$. Our main result is the following:

Theorem 1.1. *If $N \geq 3$, there exist at least two exponents $p_1, p_2 \in (1, +\infty)$ such that (p_k, u_{p_k}) is a nonradial bifurcation point for the curve \mathcal{S} , related by (1.4) to $k = 1$ and $k = 2$ at which the continuum of bifurcating solutions $\mathcal{C}(p_k)$ is unbounded in $(1, +\infty) \times C_0^{1,\alpha}(\overline{A})$.*

The unboundedness of the bifurcating branch in Theorem 1.1 implies or that the branch exists for every $p > p_k$ (giving a multiplicity result for problem (1.1)) or that the solutions along the branch make blow-up in the $C^{1,\alpha}$ -norm at some exponent $\bar{p} \geq \frac{N+2}{N-2}$. Since problem (1.1) can be supercritical we cannot exclude that the branch exists only for a fixed value of the exponent p , even if we do not think this is the case. So we think that the behavior of the solutions along these unbounded branches deserves to be investigated further.

This result is a first attempt to separate all the branches generated by spherical harmonics ($O(N-1)$ -invariant), related by (1.5) to a different eigenvalue $\mu_k = k(N - 2 + k)$ of the Laplace Beltrami operator on the $(N-1)$ -dimensional sphere. Theorem 1.1 says that the branches generated by the spherical harmonics corresponding to $\mu_1 = N - 1$ and $\mu_2 = 2N$ ($k = 1, 2$) are separated from the others. The proof relies

on the fact that functions which are $O(N-1)$ -invariant, can be written as functions which depend only on r and θ in radial coordinates, see the Appendix for details. Then to separate the continuum $\mathcal{C}(p_1)$ and $\mathcal{C}(p_2)$ from the others we introduce two different cones in $C_0^{1,\alpha}(\overline{A})$ and we set our problem on these cones.

The cones \mathcal{K}^1 and \mathcal{K}^2 are defined in Section 3, see (3.1) and (3.2) and their definition is completely new. To define \mathcal{K}^1 we look for solutions which are non increasing with respect to the angle θ on the interval $[0, \pi]$. This property is preserved along the branch $\mathcal{C}(p_1)$, as proved in Section 3, and allows to distinguish the branch generated by $k = 1$ from the others. Functions with this type of symmetry are said Foliated Schwarz symmetric and arise when looking for solutions with low Morse index (see [GPW] as an example). Indeed in our case they arise when the Morse index of the radial solution u_p goes from 1 to $N + 1$, see (2.4).

To define \mathcal{K}^2 we consider $O(N-1)$ invariant functions which are non increasing with respect to the angle θ on the interval $[0, \frac{\pi}{2}]$ and which are even in $z = \cos \theta$. Again this property is preserved along the continuum $\mathcal{C}(p_2)$ and it is enough to exclude the branches bifurcating from exponents related by (1.3) to μ_k with $k \neq 2$.

As said before this is a first attempt to separate branches of nonradial solutions when $N \geq 3$. The definition of the cones \mathcal{K}^1 and \mathcal{K}^2 is suggested by the shape of the $O(N-1)$ -invariant spherical harmonics given explicitly in the Appendix. We believe that it should be possible to separate all the branches generated by different spherical harmonics investigating in a deeper way their properties.

The cones \mathcal{K}^1 and \mathcal{K}^2 , introduced here, separate the first spherical harmonics and can be used to distinguish solutions in a radially symmetric domain. The same method can be used to separate branches of nonradial, $O(N-1)$ -invariant functions in other settings, for example in the case of the exterior of the ball, see [GP] or in the case of the critical Hénon problem in \mathbb{R}^N , see [GGN1]. We believe that also in these cases the cones \mathcal{K}^1 and \mathcal{K}^2 can give the unboundedness of the bifurcating branch of non radial solutions. Another application of these cones can be, for instance, to reduce the dimension of the kernel of the linearized equation to some problems, see [GGT] as an example.

The problem to separate branches of solutions generated by different values of k was solved in dimension $N = 2$ by Dancer in the paper [DA1]. Using the fact that the spherical harmonic functions associated to the eigenvalues μ_k are periodic with period $\frac{2\pi}{k}$ when $N = 2$ Dancer introduced some suitable cones \mathcal{K}^k of periodic functions in which only the k -th spherical harmonic lies. This allows to separate branches of solutions related to different values of μ_k and can give also multiplicity results, see [GGN2] as an example.

Using exactly the same cones \mathcal{K}^n of Dancer we have the following result:

Corollary 1.2. *If $N = 2$, for any $n \in \mathbb{N}$ there exists an exponent $p_n \in (1, +\infty)$ such that the continuum $\mathcal{C}(p_n)$ is unbounded in $C_0^{1,\alpha}(\overline{A})$. Moreover $\mathcal{C}(p_n) \cap \mathcal{C}(p_m) = \emptyset$ if $n \neq m$. Finally for any $p > p_n$ there exist at least n nonradial positive solutions of (1.1).*

The multiplicity result for $N = 2$ follows since problem (1.1) is subcritical in \mathbb{R}^2 and cannot be obtained for $N \geq 3$ in an easy way.

The paper is organized as follows: in section 2 we introduce all the notations. In section 3 we prove Theorem 1.1 and Corollary 1.2. Finally in the Appendix we derive some properties of functions which are $O(N - 1)$ -invariant and we write explicitly the $O(N - 1)$ -invariant spherical harmonics.

2. NOTATIONS AND PRELIMINARY RESULTS

The starting point in the study of bifurcation is the analysis of the degeneracy points to (1.1). To this end we consider the linearized equation at the radial solution u_p , i.e.

$$(2.1) \quad \begin{cases} -\Delta v - pu_p^{p-1}v - \lambda v = 0 & \text{in } A \\ v = 0 & \text{on } \partial A. \end{cases}$$

It is proved in [GGPS] (see Lemma 2.3) that equation (2.1) admits a nontrivial solution if and only if

$$(2.2) \quad \alpha_1(p) + \mu_k = 0, \quad \text{for some } k \geq 1,$$

where $\alpha_1(p)$ is the first eigenvalue of the one-dimensional operator \widehat{L}_p defined in (1.4) and $\mu_k = k(N - 2 + k)$, $k = 0, 1, \dots$ are the eigenvalues of the Laplace-Beltrami operator $-\Delta_{S^{N-1}}$ on the sphere S^{N-1} .

Moreover the solutions v of the linearized equation (2.1) corresponding to a degeneracy point p_i can be written as

$$(2.3) \quad v(x) = w_{1,p_i}(|x|)\phi_k\left(\frac{x}{|x|}\right)$$

where $w_{1,p_i}(r)$ is the first positive eigenfunction of \widehat{L}_{p_i} and ϕ_k is an eigenfunction of the Laplace-Beltrami operator on S^{N-1} relative to the eigenvalue μ_k .

As explained in [GGPS], the Morse index of the radial solution u_p that we denote by $m(p)$ depends only on the sign of the sum $\alpha_1(p) + \mu_k$ for $k \geq 1$ and precisely is given by (recall that $\alpha_1(p) < 0$ for any p)

$$(2.4) \quad m(p) = \sum_{\substack{0 \leq j < \frac{2-N}{2} + \frac{1}{2}\sqrt{(N-2)^2 - 4\alpha_1(p)} \\ j \text{ integer}}} \frac{(N + 2j - 2)(N + j - 3)!}{(N - 2)!j!}.$$

In [GGPS] we proved the following result:

Theorem 2.1. *The Morse index changing points are nonradial bifurcation points for (1.1). Moreover the exponents p_i of these points can be arranged in a sequence that diverges to $+\infty$.*

To introduce the global bifurcation result obtained in [G] we let X be the subspace of $C_0^{1,\alpha}(\bar{A})$ given by the functions which are $O(N - 1)$ -invariant, i.e.

$$(2.5) \quad X := \{v \in C_0^{1,\alpha}(\bar{A}) \text{ s.t. } v(x_1, \dots, x_N) = v(g(x_1, \dots, x_{N-1}), x_N) \\ \text{for any } g \in O(N - 1)\}$$

where $O(N-1)$ is the orthogonal group in \mathbb{R}^{N-1} , and

$$(2.6) \quad \begin{aligned} T(p, v) : (1, +\infty) \times X &\rightarrow X \\ (p, v) &\mapsto (-\Delta - \lambda)^{-1} (|v|^{p-1}v). \end{aligned}$$

T is a compact operator for fixed p and is continuous with respect to p . We let $S(p, v) := v - T(p, v)$. Then any solution of (1.1) can be found as a solution of $S(p, v) = 0$ such that $v \geq 0$ in the annulus A . Let us denote by Σ the closure in $(1, +\infty) \times X$ of the set of solutions of $S(p, v) = 0$ different from u_p , i.e.

$$(2.7) \quad \Sigma := \overline{\{(p, v) \in (1, +\infty) \times X, S(p, v) = 0, v \neq u_p\}}.$$

If $(p_k, u_{p_k}) \in \mathcal{S}$ is a nonradial bifurcation point, then $(p_k, u_{p_k}) \in \Sigma$. For $(p_k, u_{p_k}) \in \Sigma$ we will call $\mathcal{C}(p_k) \subset \Sigma$ the closed connected component of Σ which contains (p_k, u_{p_k}) and is maximal with respect to the inclusion.

In [G] we proved the following result:

Theorem 2.2. *Let (p_k, u_{p_k}) be a Morse index changing point and let $\mathcal{C}(p_k)$ as defined before. Then either*

- a) $\mathcal{C}(p_k)$ is unbounded in $(1, +\infty) \times X$, or
- b) for some $h \neq k$, (p_h, u_{p_h}) is a Morse index changing point and $(p_h, u_{p_h}) \in \mathcal{C}(p_k)$.

Now we are in position to prove our first new result.

Proposition 2.3. *Let (p_k, u_{p_k}) be a Morse index changing point and let $\mathcal{C}(p_k)$ be as defined before. If $\mathcal{C}(p_k)$ is bounded, the number of the Morse index changing points in $\mathcal{C}(p_k)$ including (p_k, u_{p_k}) is even.*

This result is based on an improved version of the Rabinowitz alternative due to Ize (see [N]). We report the proof for completeness, see also [AG].

Proof. If $\mathcal{C}(p_k)$ is bounded then b) of Theorem 2.2 holds and $\mathcal{C}(p_k)$ must meet the curve \mathcal{S} at least in one point (p_h, u_{p_h}) such that p_h is a degeneracy point, i.e. satisfies (2.2). But it can meet the curve \mathcal{S} also in other bifurcation points. Since $\mathcal{C}(p_k)$ is bounded and the exponents p_i of the bifurcation points must satisfy (2.2) then $\mathcal{C}(p_k)$ can meet \mathcal{S} at most in finitely many bifurcation points (p_i, u_i) , $i = 1, \dots, n$ with $p_1 < p_2 < \dots < p_n$. Arguing as in the proof of Theorem 3.3 in [G], we can find a bounded open set $\mathcal{O} \subset (1, +\infty) \times X$ such that $\mathcal{C}(p_k) \subset \mathcal{O}$, $\partial\mathcal{O} \cap \Sigma = \emptyset$ where Σ is as defined in (2.7). Moreover we can assume that \mathcal{O} does not contain points (p, u_p) if $|p - p_i| \geq \epsilon_0$ for $i = 1, \dots, n$ and $\epsilon_0 > 0$ such that there are not degeneracy points in $\cup_{i=1}^n (p_i - 2\epsilon_0, p_i + 2\epsilon_0)$. For \mathcal{O} as above and $r > 0$, consider the map

$$\begin{aligned} S_r(p, v) : \bar{\mathcal{O}} &\rightarrow X \times \mathbb{R} \\ (p, v) &\mapsto (S(p, v), \|v - u_p\|_X^2 - r^2) \end{aligned}$$

where $\|\cdot\|_X$ stands for the usual norm in the space $C_0^{1,\alpha}(A)$. Now, $\deg(S_r(p, v), \mathcal{O}, (0, 0))$ is defined since on $\partial\mathcal{O}$ there are no solutions of $S(p, v) = 0$ different from the radial solution u_p , and hence $0 = \|v - u_p\|_X < r$ for such any solution. Furthermore the degree is independent of $r > 0$. For large r , $S_r(p, v) = (0, 0)$ has no solutions in \mathcal{O} ,

and hence has degree zero. On the other hand, for small r , if (p, v) is a solution of $S_r(p, v) = (0, 0)$, then $\|v - u_p\|_X = r$, and hence p is close to one of the p_i , $i = 1, \dots, n$. But then the sum of local degrees of S_r in the neighborhoods of each of the p_i is equal to zero, so that

$$(2.8) \quad 0 = \sum_{i=1}^n \deg(S_r(p, v), \mathcal{O} \cap B_r(p_i, u_{p_i}), (0, 0)).$$

In particular we choose $r < \epsilon_0$ for ϵ_0 defined as before. In order to compute the degree of $S_r(p, v)$ in $\mathcal{O} \cap B_r(p_i, u_{p_i})$ we use again the homotopy invariance of the degree. Let us define

$$S_r^t(p, v) = (S(p, v), t(\|v - u_p\|_X^2 - r^2) + (1 - t)(2p_i p - p^2 - p_i^2 + r^2))$$

for $t \in [0, 1]$. As before $\deg(S_r^t(p, v), \mathcal{O} \cap B_r(p_i, u_{p_i}), (0, 0))$ is well defined since there are no solutions on the boundary if r is small (recall that $u_{p_i \pm r}$ are isolated if $r < \epsilon_0$). Moreover the degree is independent of t . For $t = 1$ we have $S_r^1(p, v) = S_r(p, v)$, while for $t = 0$, $S_r^0(p, v) = (S(p, v), 2p_i p - p^2 - p_i^2 + r^2)$ and

$$\begin{aligned} & \deg(S_r^0(p, v), \mathcal{O} \cap B_r(p_i, u_{p_i}), (0, 0)) \\ &= \deg(S(p, v), \mathcal{O} \cap B_r(p_i, u_{p_i}), 0) \cdot \deg(2p_i p - p^2 - p_i^2 + r^2, \{|p - p_i| < r\}, 0). \end{aligned}$$

Now

$$\deg(2p_i p - p^2 - p_i^2 + r^2, \{|p - p_i| < r\}, 0) = 1$$

for $p = p_i - r$ while

$$\deg(2p_i p - p^2 - p_i^2 + r^2, \{|p - p_i| < r\}, 0) = -1$$

for $p = p_i + r$. This implies that

$$\begin{aligned} & \deg(S_r(p, v), \mathcal{O} \cap B_r(p_i, u_{p_i}), (0, 0)) = \\ & \deg(S(p_i - r, \cdot), \mathcal{O}_{p_i - r}, 0) - \deg(S(p_i + r, \cdot), \mathcal{O}_{p_i + r}, 0) \\ &= (-1)^{m(p_i - r)} - (-1)^{m(p_i + r)} \end{aligned}$$

where as in [G] we denote by \mathcal{O}_p the set $\{v \in \mathcal{O} : (p, v) \in \mathcal{O}\}$.

We conclude that if (p_i, u_{p_i}) is a Morse index changing point then

$$\deg(S_r(p, v), \mathcal{O} \cap B_r(p_i, u_{p_i}), (0, 0)) = \pm 2$$

while if (p_i, u_{p_i}) is not a Morse index changing point then

$$\deg(S_r(p, v), \mathcal{O} \cap B_r(p_i, u_{p_i}), (0, 0)) = 0.$$

Since the nonzero terms in (2.8) correspond only to the Morse index changing points, and since these terms add up to zero, there must be an even number of Morse index changing points. \square

We let (p_k, u_{p_k}) be a bifurcation point, corresponding, via (1.3) to the eigenvalue μ_k . Then the linearized operator L_{p_k} at u_{p_k} has, up to a constant multiple, a unique solution in X , see [SW1], which has the form given by (2.3). We let w_k be this unique normalized (in the L^∞ -norm) eigenfunction of $L_{p_k}(v) = 0$ in X . Now we want to prove the following result:

Proposition 2.4. *There exists $\rho_0 > 0$ such that if $(p, v) \in (\mathcal{C}(p_k) \setminus \{(p_k, u_{p_k})\}) \cap B_\rho(p_k, u_{p_k})$, then $v - u_p = \alpha_p w_k + r_p$, where w_k is as before and $\alpha_p \rightarrow 0$ as $p \rightarrow p_k$ and $r_p = o(\alpha_p)$ as $p \rightarrow p_k$.*

Proof. With the previous notations we let l_k be an element of the dual space X' of X , such that $\langle l_k, w_k \rangle = 1$, where $\langle \cdot, \cdot \rangle$ denotes the duality between X and X' . This element l_k exists thanks to the Hahn-Banach Theorem. Let $X_k = \{z \in X \text{ such that } \langle l_k, z \rangle = 0\}$. Since $\dim(\text{Ker } L_{p_k})$ is finite, then $\text{Ker } L_{p_k}$ is complemented in X , see for example [M] pag 300. Hence we can decompose $X = \mathbb{R} \oplus X_k$, and every $z \in X$ can be written as $z = \alpha w_k + r$, with $\alpha = \langle l_k, z \rangle$ and $r \in X_k$.

For $\eta \in (0, 1)$ and $\gamma > 0$, we let

$$K_{\gamma, \eta} = \{(p, v) \in X \text{ such that } |p - p_k| < \gamma \text{ and } |\langle l_k, v - u_p \rangle| > \eta \|v - u_p\|_\infty\}$$

where $\|\cdot\|_\infty$ denotes the usual L^∞ -norm. We want to prove, first, that, for any η and any γ , there exists $\rho_0 > 0$ such that for all $\rho < \rho_0$ we have $(\mathcal{C}(p_k) \setminus \{(p_k, u_{p_k})\}) \cap B_\rho(p_k, u_{p_k}) \subset K_{\gamma, \eta}$. Here $B_\rho(p_k, u_{p_k})$ denotes the ball of radius ρ in the product space $(1, +\infty) \times X$. If there is not such a ρ_0 , there exist sequences $\rho_n \rightarrow 0$ and $(p_n, v_n) \in (\mathcal{C}(p_k) \setminus \{(p_k, u_{p_k})\}) \cap B_{\rho_n}(p_k, u_{p_k})$ such that $|p_n - p_k| \leq \rho_n < \gamma$, $v_n - u_{p_n} \rightarrow 0$ in X and $|\alpha_n| := |\langle l_k, v_n - u_{p_n} \rangle| \leq \eta \|v_n - u_{p_n}\|_\infty$. Letting $z_n = \frac{v_n - u_{p_n}}{\|v_n - u_{p_n}\|_\infty}$ we have that z_n satisfies

$$(2.9) \quad \begin{cases} -\Delta z_n = h_n(x) z_n + \lambda z_n & \text{in } A \\ z_n = 0 & \text{on } \partial A \end{cases}$$

where

$$(2.10) \quad h_n(x) = p_n \int_0^1 (u_{p_n} + t(v_n - u_{p_n}))^{p_n-1} dt$$

and $h_n(x) \rightarrow p_k u_{p_k}^{p_k-1}$ in X as $n \rightarrow +\infty$. Then $\|z_n\|_\infty = 1$ and $z_n \rightarrow z$ uniformly in \bar{A} where z is a solution of

$$(2.11) \quad \begin{cases} -\Delta z = p_k u_{p_k}^{p_k-1} z + \lambda z & \text{in } A \\ z = 0 & \text{on } \partial A \end{cases}$$

such that $\|z\|_\infty = 1$. This implies that either $z = w_k$ or $z = -w_k$. Moreover $\frac{\alpha_n}{\|v_n - u_{p_n}\|_\infty} := \frac{|\langle l_k, v_n - u_{p_n} \rangle|}{\|v_n - u_{p_n}\|_\infty} \rightarrow |\langle l_k, \pm w_k \rangle| = 1 > \eta$ and we get a contradiction. Thus there exists a $\rho_0 > 0$ as above. Using the previous decomposition $X = \mathbb{R} \oplus X_k$, we have that $v - u_p = \alpha_p w_k + r_p$ where $\alpha_p := \langle l_k, v - u_p \rangle$ and $r_p := v - u_p - \alpha_p w_k \in X_k$.

Now, if $p_n \rightarrow p_k$ then $|\alpha_n| := |\alpha_{p_n}| = |\langle l_k, v_n - u_{p_n} \rangle| = \|v_n - u_{p_n}\|_\infty |\langle l_k, z_n \rangle| = \|v_n - u_{p_n}\|_\infty (1 + o(1))$. This implies that $\alpha_n \rightarrow 0$ as $p_n \rightarrow p_k$. Finally $r_n := r_{p_n} = v_n - u_{p_n} - \alpha_n w_k$ so that

$$\begin{aligned} \|r_n\|_\infty &\leq \|v_n - u_{p_n}\|_\infty + |\alpha_n| \|w_k\|_\infty < \frac{1}{\eta} |\langle l_k, v_n - u_{p_n} \rangle| + |\alpha_n| \\ &= |\alpha_n| + \frac{1}{\eta} |\alpha_n| |\langle l_k, w_k \rangle| + \frac{1}{\eta} |\langle l_k, r_n \rangle| = \frac{1}{\eta} |\alpha_n| + |\alpha_n|. \end{aligned}$$

This shows that $r_n \rightarrow 0$ as $p_n \rightarrow p_k$. Finally

$$\left| \langle l_k, \frac{u_n - u_{p_n}}{\|v_n - u_{p_n}\|_\infty} \rangle \right| = \frac{\alpha_n}{\|v_n - u_{p_n}\|_\infty} \rightarrow 1$$

so that

$$\frac{r_n}{\|v_n - u_{p_n}\|_\infty} = \frac{v_n - u_{p_n} - \alpha_n w_k}{\|v_n - u_{p_n}\|_\infty} \rightarrow w_k - w_k = 0.$$

This shows that $r_n = o(|\alpha_n|)$ as $p_n \rightarrow p_k$ and finishes the proof. \square

This proposition gives us the behavior of the branch of solutions $\mathcal{C}(p_k)$ near a bifurcation point (p_k, u_{p_k}) .

3. PROOF OF THE MAIN RESULT.

In this section we want to prove Theorem 1.1. As before we consider functions which are $O(N-1)$ -invariant, i.e. the space X defined in (2.5). It is easy to see (see the Appendix for the details) that if $(\rho, \phi_1, \dots, \phi_{N-2}, \theta)$ with $a \leq \rho \leq b$ and $\phi_i \in [0, 2\pi]$ for $i = 1, \dots, N-2$ and $\theta \in [0, \pi]$, are the radial coordinates in \mathbb{R}^N , then a $O(N-1)$ -invariant function in \mathbb{R}^N can be written as a function which depends only on ρ and θ .

We introduce the following cones:

$$(3.1) \quad \mathcal{K}^1 = \left\{ \begin{array}{l} v \in C^{1,\alpha}(\bar{A}) \text{ s.t. } v \text{ is } O(N-1) \text{ - invariant, } v \geq 0 \text{ in } A, \\ v = v(\rho, \theta) \text{ in radial coordinates for } (\rho, \theta) \in A, \\ v(\rho, \theta) \text{ is even in } \theta \text{ and } v(\rho, \pi + \theta) = v(\rho, \pi - \theta) \\ v(\rho, \theta) \text{ is non increasing in } \theta \text{ for } \theta \in [0, \pi], \rho \in [a, b] \end{array} \right\}$$

and

$$(3.2) \quad \mathcal{K}^2 = \left\{ \begin{array}{l} v \in C^{1,\alpha}(\bar{A}) \text{ s.t. } v \text{ is } O(N-1) \text{ - invariant, } v \geq 0 \text{ in } A, \\ v = v(\rho, \theta) \text{ in radial coordinates for } (\rho, \theta) \in A, \\ v(\rho, z) \text{ is even in } z, \text{ where } z = \cos \theta, \text{ for } z \in [-1, 1] \\ v(\rho, \theta) \text{ is non increasing in } \theta \text{ for } \theta \in [0, \frac{\pi}{2}], \rho \in [a, b] \end{array} \right\}.$$

As said in the Introduction functions that belong to \mathcal{K}^1 are Foliated Schwarz symmetric, see [GPW] for some comments on this type of symmetry.

Since the terminology is not uniform in the literature we recall that W is a cone in X , if W is a closed convex set in X such that $\gamma W \subseteq W$ for any $\gamma \geq 0$ and $W \cap -W = \{\emptyset\}$.

First we can prove the following result:

Lemma 3.1. *For any $p \in (1, +\infty)$ the map $T(p, -) : X \rightarrow X$, defined in (2.6), maps the cone \mathcal{K}^i into itself.*

Proof. Suppose $g \in \mathcal{K}^i$, then the function $g^p \in \mathcal{K}^i$ for $i = 1, 2$. We have that $T(p, g) = w$ if w is a solution to

$$(3.3) \quad \begin{cases} -\Delta w - \lambda w = g^p & \text{in } A \\ w = 0 & \text{on } \partial A. \end{cases}$$

First, since $g \geq 0$ in A and $\lambda \leq 0$, the Maximum principle implies $w \geq 0$ in A . We already know that the map $T(p, -)$ is invariant with respect the action of the

group $O(N-1)$, so that $T(p, -)$ maps the space X into itself. Thus w is $O(N-1)$ -invariant and, as previously said, we can write $w = w(\rho, \theta)$ with the properties $w(\rho, \theta) = w(\rho, -\theta)$ and $w(\rho, \pi + \theta) = w(\rho, \pi - \theta)$ for every $(\rho, \theta) \in A$, see the Appendix for these details. Moreover, if g is even in z then g^p is even in z and so also $w = T(p, g)$ is even in z . Exploiting this fact we can rewrite (3.3) in radial coordinates, getting that w satisfies

$$(3.4) \quad \begin{cases} -\frac{\partial^2 w}{\partial \rho^2} - \frac{N-1}{\rho} \frac{\partial w}{\partial \rho} - \frac{1}{\rho^2} \frac{\partial^2 w}{\partial \theta^2} - \frac{N-2}{\rho^2} \cot \theta \frac{\partial w}{\partial \theta} - \lambda w = g^p & \text{in } A \\ w = 0 & \text{on } \partial A \end{cases}$$

Differentiating with respect to θ we get that $w_\theta := \frac{\partial w}{\partial \theta}$ satisfies

$$-\frac{\partial^2 w_\theta}{\partial \rho^2} - \frac{1}{\rho^2} \frac{\partial^2 w_\theta}{\partial \theta^2} - \frac{N-1}{\rho} \frac{\partial w_\theta}{\partial \rho} - \frac{N-2}{\rho^2} \cot \theta \frac{\partial w_\theta}{\partial \theta} + \frac{N-2}{\rho^2 \sin^2 \theta} w_\theta - \lambda w_\theta = p g^{p-1} \frac{\partial g}{\partial \theta}$$

for $\rho \in [a, b]$ and $\theta \in [0, \pi]$. This is a second order operator uniformly elliptic in $[a, b] \times [0, \pi]$. The coefficient of the linear term is $c(\rho, \theta) = \frac{N-2}{\rho^2 \sin^2 \theta} - \lambda > 0$ in $(a, b) \times (0, \pi)$ and it is bounded in every closed ball in $(a, b) \times (0, \pi)$. Also the coefficients of the first order terms, i.e. $\frac{N-1}{\rho}$ and $\frac{N-2}{\rho^2} \cot \theta$ are bounded on every closed ball in $(a, b) \times (0, \pi)$. Then, we consider first the case of $g \in \mathcal{K}^1$, the maximum principle applies since $\frac{\partial g}{\partial \theta} \leq 0$ for $(\rho, \theta) \in (a, b) \times (0, \pi)$ and implies that w_θ reaches its maximum on the boundary of $(a, b) \times (0, \pi)$, see [PW] pag 64. Then, the boundary conditions $v(a, \theta) = v(b, \theta) = 0$ for every θ imply that $w_\theta(a, \theta) = w_\theta(b, \theta) = 0$ for every $\theta \in [0, \pi]$. Finally the symmetry assumptions on v imply, in turn, that $w_\theta(\rho, 0) = w_\theta(\rho, \pi) = 0$, see the Appendix for details, so that $w_\theta \leq 0$ in $[a, b] \times [0, \pi]$. This implies that $v \in \mathcal{K}^1$ and concludes the proof in the case of $g \in \mathcal{K}^1$.

Now assume $g \in \mathcal{K}^2$. As said before $w = T(p, g)$ is even in z . By assumptions we have that $\frac{\partial g}{\partial \theta} \leq 0$ for $(\rho, \theta) \in (a, b) \times (0, \frac{\pi}{2})$. Again we can apply the maximum principle getting that w_θ reaches its maximum on the boundary of $(a, b) \times (0, \frac{\pi}{2})$. As before $w_\theta(a, \theta) = w_\theta(b, \theta) = 0$ for every $\theta \in [0, \frac{\pi}{2}]$ and $w_\theta(\rho, 0) = 0$ for every $\rho \in (a, b)$. Finally since w is even in z we get that $w(\rho, \cos \theta) = w(\rho, -\cos \theta)$ and this implies $w_\theta(\rho, \frac{\pi}{2}) = 0$ for any $\rho \in (a, b)$. Then we have $w_\theta \leq 0$ in $[a, b] \times [0, \frac{\pi}{2}]$ showing that $w \in \mathcal{K}^2$. This concludes the proof of the Lemma. \square

Before proving the main result we need some notations, following [DA]. Given a cone \mathcal{K} and a point $u \in \mathcal{K}$ we let $\mathcal{K}_u := \{v \in X : u + tv \in \mathcal{K} \text{ for some } t > 0\}$ and $S_u := \{v \in \overline{\mathcal{K}}_u : -v \in \overline{\mathcal{K}}_u\}$. Then we have that, if u_p is a radial solution of (1.1) then $\frac{\partial u_p}{\partial \theta} \equiv 0$ in A and hence $\overline{\mathcal{K}}_{u_p}^1 = \{v \in X : v = v(\rho, \theta) \text{ in radial coordinates for } (\rho, \theta) \in A, v(\rho, \theta) \text{ is even in } \theta, \frac{\partial v}{\partial \theta} \geq 0 \text{ in } A\}$ while $\overline{\mathcal{K}}_{u_p}^2 = \{v \in X : v = v(\rho, \theta) \text{ in radial coordinates for } (\rho, \theta) \in A, v(\rho, z) \text{ is even in } z = \cos \theta, \frac{\partial v}{\partial \theta} \geq 0 \text{ for any } \theta \in [0, \frac{\pi}{2}], \rho \in [a, b]\}$. This implies that, using the fact that $v \in \mathcal{K}^2$ is even in z , $S_{u_p}^1 = S_{u_p}^2 = \{v \in X : \text{such that } v \text{ is radially symmetric}\}$. See [DA1] for details.

Proposition 3.2. *Let u_p be a radial solution of (1.1) which is nondegenerate. Then for $i = 1, 2$*

$$(3.5) \quad \text{index}_{\mathcal{K}^i}(I - T(p, -), u_p) = \begin{cases} \pm 1 & \text{if } \alpha_1(p) + \mu_i > 0 \\ 0 & \text{if } \alpha_1(p) + \mu_i < 0 \end{cases}$$

where $\alpha_1(p)$ is the first eigenvalue of the radial operator defined in (1.4) and $\mu_1 = N - 1$ and $\mu_2 = 2N$.

Proof. To calculate the index of $I - T(p, -)$ in the cone \mathcal{K}^i at the radial solution u_p we use Theorem 1 in [DA]. First we observe that, with the previous notations, we have that $\mathcal{K}^i - \mathcal{K}^i$ is dense in X . Moreover by assumptions we have that u_p is a fixed point of $T(p, -)$ in \mathcal{K}^i and $T(p, u_p)$ is differentiable at u_p with $T'(p, u_p)$ invertible, since u_p is nondegenerate. We are in position to apply Theorem 1 in [DA] getting that

$$\text{index}_{\mathcal{K}^i}(I - T(p, -), u_p) = \begin{cases} 0 & \text{if } \alpha_1(p) + \mu_i < 0 \\ \text{index}_X(I - T(p, -), u_p) = \pm 1 & \text{otherwise.} \end{cases}$$

This claim follows from [DA, Theorem 1] if we check that $T'(p, u_p)$ has an eigenvalue in $(1, +\infty)$ with corresponding eigenvector in $\bar{\mathcal{K}}_{u_p}^i \setminus S_{u_p}$ if and only if $\alpha_1(p) + \mu_i < 0$, see also Lemma 2 in [DA] and the Remark after it.

This is equivalent to show that the linearized operator has a negative eigenvalue with eigenfunction in $\bar{\mathcal{K}}_{u_p}^i \setminus S_{u_p}$. In [GGPS] it is shown that the solutions of the linearized equation have the form given in (2.3). This result holds also for the eigenfunctions of the eigenvalue problem with weight associated to the linearized equation, see [GGN1] for a proof of this assertion. Then, if we restrict to the space X we have that an eigenvalue of the linearized problem becomes negative each time $\alpha_1(p) + \mu_k$ becomes negative and the corresponding eigenfunctions have the form given in (2.3), i.e. is the product of a positive radial function for a $O(N - 1)$ -invariant spherical harmonic function. Using the characterization of $O(N - 1)$ -invariant spherical harmonics we have that the linearized operator has a negative eigenvalue in \mathcal{K}^1 if and only if $\alpha_1(p) + \mu_1 < 0$, while the linearized operator has a negative eigenvalue in \mathcal{K}^2 if and only if $\alpha_1(p) + \mu_2 < 0$, since the eigenfunction corresponding to the negative eigenvalue $\alpha_1(p) + \mu_1$ does not belong to \mathcal{K}^2 (it is not even in z). This finishes the proof. \square

Proof of Theorem 1.1. We prove the result in the case of the exponent p_1 related by (1.3) to μ_1 . The case of the exponent p_2 related to μ_2 follows in the same way substituting the cone \mathcal{K}^1 with \mathcal{K}^2 .

Let p_1, \dots, p_M be the Morse index changing points related by (1.3) to the first eigenvalue μ_1 . We can repeat the proof of Theorem 3.3 in [G] using the cone \mathcal{K}^1 instead of the space X . Hence we get, for any p_j , $j = 1, \dots, M$, the existence of a continuum $\mathcal{C}(p_j)$ of solutions of (1.1) which lies in the cone \mathcal{K}^1 . Further this continuum either is unbounded in \mathcal{K}^1 or it must intersect the curve of radial solutions \mathcal{S} in another Morse index changing point. Moreover the points at which $\mathcal{C}(p_j)$

can intersect the curve of radial solutions \mathcal{S} are related to the first eigenvalue μ_1 . This follows since otherwise the continuum $\mathcal{C}(p_j)$ is not contained in \mathcal{K}^1 , see also Proposition 2.4. Repeating the proof of Proposition 2.3, in the cone \mathcal{K}^1 we have that the number of Morse index changing points which belong to a bounded continuum $\mathcal{C}(p_j)$ has to be even. On the other hand the number of Morse index changing points corresponding to the eigenvalue μ_1 , is odd, since $\alpha_1(p) + \mu_1 > 0$ if p is near 1 while $\alpha_1(p) + \mu_1 < 0$ if p is large enough.

This implies the existence of a value p_1 such that $\alpha_1(p_1) + \mu_1 = 0$ and $\mathcal{C}(p_1)$ is unbounded in X . \square

Remark 3.3. *We suspect that the equation $\alpha_1(p) + \lambda_k = 0$ has only one solution, but we are not able to prove it. In that case any degeneracy point would be a Morse index changing point and the branch of bifurcating solutions would be unbounded.*

Now we sketch the proof of Corollary 1.2. Let (ρ, θ) be the radial coordinates in \mathbb{R}^2 . As said before the proof of this result follows using the cones

$$\mathcal{K}^n := \left\{ v \in C(A) : v \geq 0 \text{ in } A, v(\rho, \theta) = v(\rho, \theta + \frac{2\pi}{n}) \text{ for } (r, \theta) \in A, v \text{ is even in } \theta, \right. \\ \left. v(\rho, \theta) \text{ is decreasing in } \theta \text{ for } 0 < \theta < \frac{\pi}{n}, a \leq \rho \leq b \text{ and } v = 0 \text{ on } \partial A \right\}$$

introduced by Dancer in [DA1]. For these cones \mathcal{K}^n the analogous of Lemma 1 and Theorem 1 in [DA1] holds. These cones allow to separate continua of solutions of (1.1) in \mathbb{R}^2 related by (1.3) to a different eigenvalue μ_k . Using, as in the proof of Theorem 1.1, the Proposition 2.3 then we have that, corresponding to any k , there exists at least a continuum $\mathcal{C}(p_k)$ which is unbounded in X . Finally since we are in dimension 2 the solutions of (1.1) cannot blow up at a finite value p^* , so that the unbounded continuum $\mathcal{C}(p_k)$ has to be defined for every $p > p_k$.

4. APPENDIX

The $O(N-1)$ -invariant functions in \mathbb{R}^N .

Let us consider the spherical coordinates in \mathbb{R}^N , $(\rho, \phi_1, \dots, \phi_{N-2}, \theta)$ where $\phi_i \in [0, 2\pi]$, $i = 1, \dots, N-2$ and $\theta \in [0, \pi]$. As usual

$$\begin{cases} x_i = \rho \sin \theta H_i(\phi_1, \dots, \phi_{N-2}) & i = 1, \dots, N-2 \\ x_N = \rho \cos \theta \end{cases}$$

where H_i are suitable functions. We are interested in the $O(N-1)$ -invariant functions, i.e. functions v such that

$$v(x_1, \dots, x_N) = v(g(x_1, \dots, x_{N-1}), x_N)$$

for any $g \in O(N-1)$. By definition, a function which is $O(N-1)$ -invariant depends only on $\rho' = \sqrt{x_1^2 + \dots + x_{N-1}^2}$ and x_N . Then, in radial coordinates, since $\rho' = \sqrt{\rho^2 - x_N^2} = \sqrt{\rho^2(1 - \cos^2 \theta)} = \rho |\sin \theta|$ and $x_N = \rho \cos \theta$, v can be written as a function which depends only on ρ and θ .

Moreover, v must satisfy $v(\rho, \theta) = v(\rho, -\theta)$ and $v(\rho, \pi + \theta) = v(\rho, \pi - \theta)$. This

assertion follows since v must depend only on ρ' and x_N and as functions of ρ, θ they satisfy $\rho'(\rho, -\theta) = \rho'(\rho, \theta)$, $x_N(\rho, -\theta) = x_N(\rho, \theta)$ and $\rho'(\rho, \pi + \theta) = \rho|\sin(\pi + \theta)| = \rho|\sin \theta| = \rho|\sin(\pi - \theta)| = \rho'(\rho, \pi - \theta)$, $x_N(\rho, \pi + \theta) = \rho \cos(\pi + \theta) = -\rho \cos \theta = \rho \cos(\pi - \theta) = x_N(\rho, \pi - \theta)$.

Then an $O(N-1)$ -invariant function v satisfies $v(\rho, \theta) = v(\rho, -\theta)$ and $v(\rho, \pi + \theta) = v(\rho, \pi - \theta)$ and, if $v \in C^1(A)$ then it is $C^1((a, b) \times [0, \pi])$ and it verifies $\frac{\partial v}{\partial \theta}(\rho, 0) = \frac{\partial v}{\partial \theta}(\rho, \pi) = 0$ for any $\rho > 0$.

Some remarks on the $O(N-1)$ -invariant spherical harmonic functions.

From what we said before the $O(N-1)$ -invariant spherical harmonics can be written as functions which depend only on the variable θ . Then, the k -th $O(N-1)$ -invariant spherical harmonic satisfies

$$(4.1) \quad -\sin^2 \theta \frac{\partial^2 \Phi_k}{\partial \theta^2} - (N-2) \sin \theta \cos \theta \frac{\partial \Phi_k}{\partial \theta} = \lambda_k \sin^2 \theta \Phi_k$$

for $\theta \in (0, \pi)$. Letting $z = \cos \theta$ we get that $\Phi_k(z)$ satisfies

$$(4.2) \quad (1-z^2) \frac{\partial^2 \Phi_k}{\partial z^2} - (N-1)z \frac{\partial \Phi_k}{\partial z} + \lambda_k \Phi_k = 0$$

for $z \in (-1, 1)$. This is a *Sturm-Liouville problem*. Then we can say that the k -th eigenfunction has k different zeros in $[-1, 1]$. From the Sturm Theorem between two consecutive zeros of Φ_k there is a zero of Φ_{k+1} .

Equation (4.2) is the *Jacobi equation* with, using the usual notations for Jacobi, $\alpha = \beta = \frac{N-3}{2}$ and $n = k$. Then the bounded solutions of (4.2) are given, up to a constant multiple, by the Jacobi polynomials, that can be written, using the Rodrigues' formula

$$(4.3) \quad P_k^{(\frac{N-3}{2}, \frac{N-3}{2})}(z) = \frac{(-1)^k}{2^k k!} (1-z^2)^{-\frac{N-3}{2}} \frac{\partial^k}{\partial z^k} \left((1-z^2)^{k+\frac{N-3}{2}} \right)$$

for $z \in (-1, 1)$ and any $k \geq 0$.

If $\alpha = \beta = 0$, i.e. for $N = 3$, the Jacobi polynomials reduce to the Legendre polynomials

$$P_k(z) = \frac{1}{2^k k!} \frac{\partial^k}{\partial z^k} (z^2 - 1)^k$$

and, indeed for $N = 3$, (4.2) is the classical Legendre equation.

Then the $O(N-1)$ -invariant spherical harmonics are, up to a constant multiple, the functions:

$$\Phi_k(\theta) = P_k^{(\frac{N-3}{2}, \frac{N-3}{2})}(\cos \theta)$$

for $\theta \in (0, \pi)$, where $P_k^{(\frac{N-3}{2}, \frac{N-3}{2})}$ are the Jacobi Polynomials.

To give some examples we have

$$\begin{aligned} \Phi_1(\theta) &= \frac{N-1}{2} \cos \theta, \\ \Phi_2(\theta) &= \frac{N-1}{8} (N \cos^2 \theta - 1), \\ \Phi_3(\theta) &= \frac{1}{48} (N+3)(N+1) ((N+2) \cos^3 \theta - 3 \cos \theta) \end{aligned}$$

This implies, in turn, that the unique $O(N-1)$ -invariant spherical harmonic Φ_1 related to the first eigenvalue λ_1 is, up to a constant multiple,

$$\Phi_1(\theta) = \cos \theta.$$

It then follows that $\frac{\partial \Phi_1}{\partial \theta} = -\sin \theta \leq 0$ in $[0, \pi]$ while for all the other spherical harmonics the derivative $\frac{\partial \Phi_i}{\partial \theta}$ must change sign in $[0, \pi]$. This can be seen since from the formulation (4.2) $\Phi_1(z) = z$ changes sign once in $(-1, 1)$ so that any other solution $\Phi_i(z)$ changes sign at least twice in $(-1, 1)$ and this implies that the derivative $\frac{\partial \Phi_i}{\partial z}$ has to change sign in $(-1, 1)$ so that also $\frac{\partial \Phi_i}{\partial \theta}$ has to change sign in $(0, \pi)$.

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